Karlsruher Institut für Technologie

Section 1.3

INTERACTION METHODS IN VACUUM TUBES

Literatur Chapter 1.3

Shulim E. Tsimring, **Electron Beam and Microwave Vacuum Electronics**,
John Wiley & Sons, New Jersey 2007

D. M. Pozar, **Microwave Engineering,**
John Wiley & Sons, New Jersey 2011

Gilmour, A. S., Jr., **Microwave Tubes**,
Artech House, Norwood, MA, 1986

Energy Transformation in Electron Ray Tubes

Basically applies:

In an electron ray tube arise the electro magnetic field (an electro magnetic wave) by a transformation of the electron kinetic energy into electro magnetic energy.

An electron, which moves in the vacuum with a constant velocity, emits no electromagnetic wave.

In order to produce an electro magnetic radiation, the electron have to suffer an acceleration (braking), or rather have to move in a medium where the electron propagation speed is higher than the propagation speed of the interactive electro magnetic wave.

The Classification of the Energy Supply to the Electromagnetic Field (Photon)

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The disposal of the kinetic energy from a relativistic electron beam to an electro magnetic field can be divided into the following 3 interaction mechanisms:

- **Tscherenkow radiation** („cherenkov radiation“, „cerenkov radiation“)
- **Transition radiation**
- **Bremsstrahlung radiation**

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- geladenes Teilchen mit v > c/n

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PSI - Die Physik-Schülerlabor-Initiative

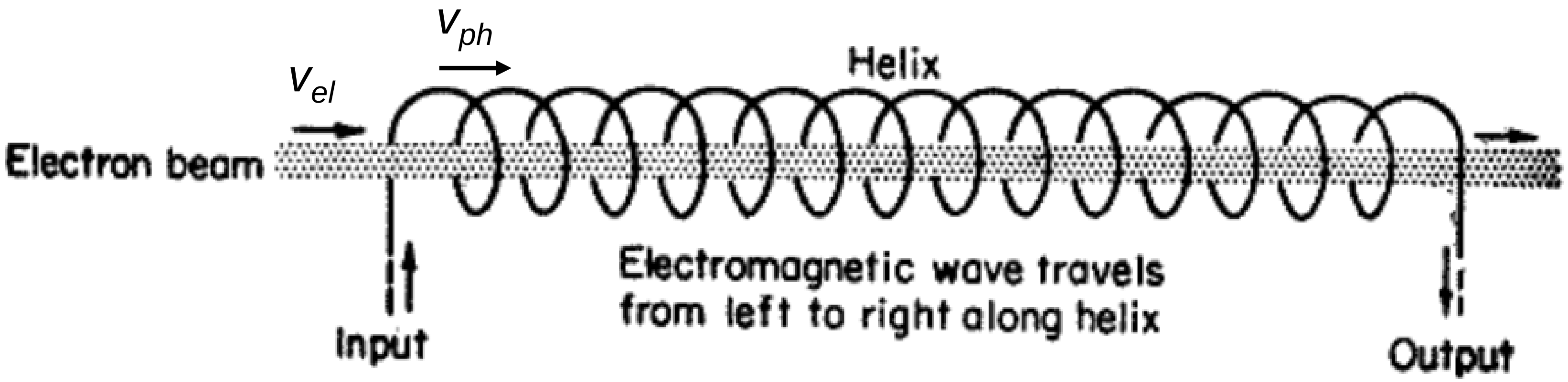
$$n := \frac{c}{v} \text{ refraction index}$$

Cherenkov Radiation in Vacuum Electron Tubes (here TWT)

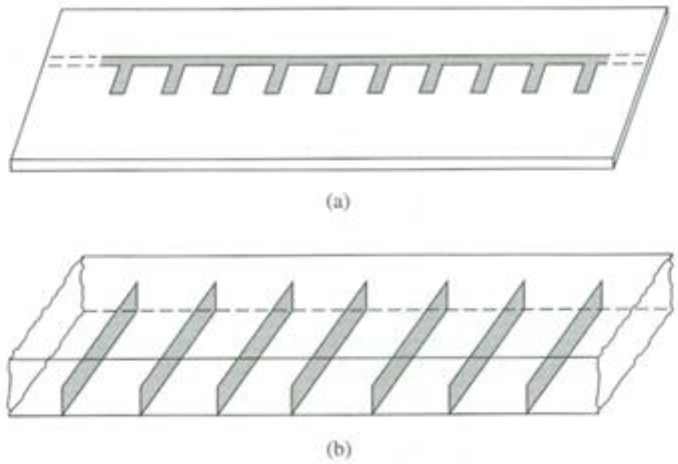
Cherenkov radiation is generated when a charged particle moves with a speed v_{el} that is larger than the phase velocity v_{ph} of electromagnetic (RF) waves in the same medium.

In general, each particle radiates independently. Therefore, the radiation is non-coherent and not effective for radio electronics.

For the radiation to become coherent, bunches have to be formed. This is possible when the difference v_{el} - v_{ph} is not large. Then bunching is forced, and the radiation is not spontaneous but induced by the RF field.



Excursion: Periodic Structures



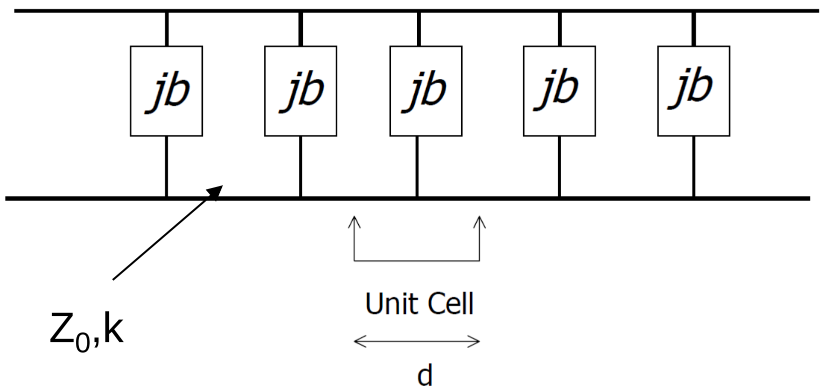
Example of periodic structures. (a) Periodic stubs on a microstrip line. (b) Periodic diaphragms in a waveguide.

D. M. Pozar,
„Microwave Engineering“, pp. 380,
Wiley&Sons, 2011

Every periodic structure can be represented by a periodically loaded transmission line

ABCD Matrix:

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix},$$

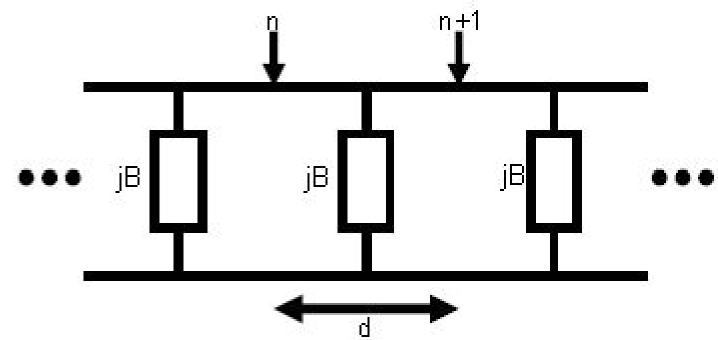


Equivalent circuit of a periodically loaded transmission line. The unloaded line has charactesitic impedance Z_0 and propagation constant k.

Transmission Line Analysis – 1 Element

Voltage and currents on either side of each cell are represented by

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}$$



$$\theta := kd$$

$$Z_0 := 1 \text{ (normalized impedance)}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\frac{\theta}{2} - \frac{b}{2}\sin\theta & j\left(\sin\theta + \frac{b}{2}\cos\theta - \frac{b}{2}\right) \\ j\left(\sin\theta + \frac{b}{2}\cos\theta + \frac{b}{2}\right) & \cos\theta - \frac{b}{2}\sin\theta \end{bmatrix}$$

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}$$

where

$$A = \cos\theta \; -(B/2)\sin\theta$$

$$B = j((B/2)\cos\theta + \sin\theta - B/2)$$

$$C = j((B/2)\cos\theta + \sin\theta + B/2)$$

$$D = \cos\theta \; -(B/2)\sin\theta$$

Transmission Line Analysis – Periodicity:

For any wave propagating in +z-direction will be valid:

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = e^{-\gamma d} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \qquad \gamma := \alpha + \mathrm{j}\beta \text{ : complex propagation constant}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} e^{-\gamma d} & 0 \\ 0 & e^{\gamma d} \end{bmatrix}$$

The above can be reduced to

$$\boxed{\cosh \gamma d = \cos\theta - (B/2)\sin\theta \qquad (4)}$$

If the magnitude of the right hand side is smaller than unity, then $\alpha = 0$ and $\gamma = j\beta$. Hence, under this condition, the periodic structure supports a propagating wave. On the other hand, if the magnitude of the right hand side of (4) is larger than unity, then no wave can propagate along the structure.

T. Itoh, „*periodic structures*“

Definition: Brillouin (k – β) -Diagram

Example: Standard waveguide

Pozar, p. 386

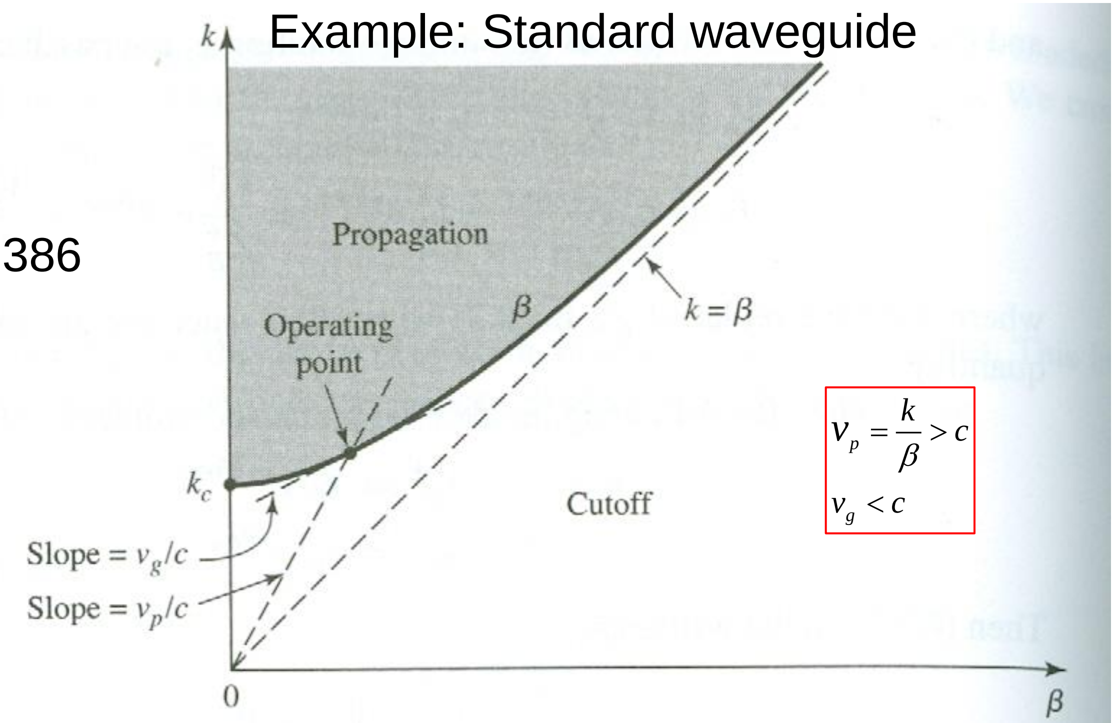


FIGURE 8.4 k-β diagram for a waveguide mode.

$$v_p = \frac{k}{\beta} > c$$
$$v_g < c$$

Dispersion relation:

$\beta := \sqrt{k^2 - k_c^2}$: propagation constant of a mode

$k := \sqrt{\beta^2 + k_c^2}$: free-space wave number

k_c : cut-off wave number

Definition: Phase- and group velocities

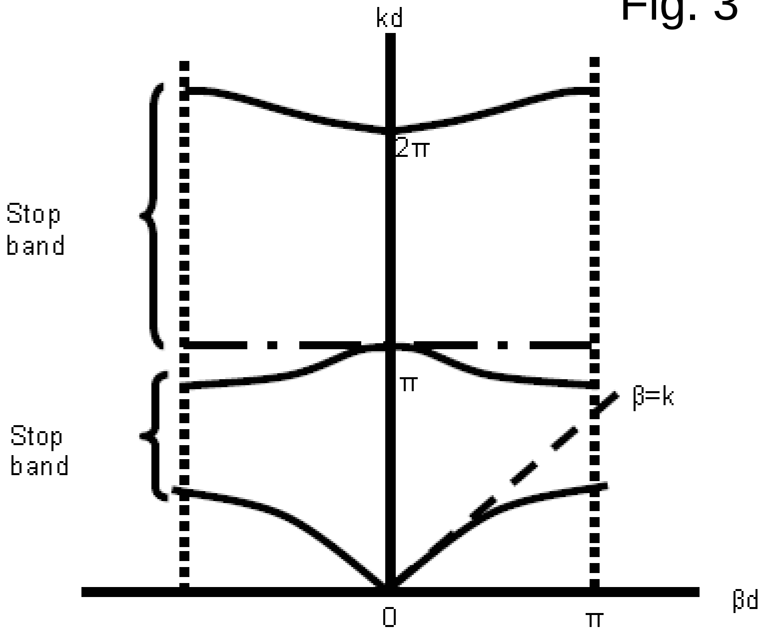
$v_p := \frac{\omega}{\beta} = c\,\frac{k}{\beta}$: phase velocity

$v_g := \frac{\mathrm{d}\omega}{\mathrm{d}\beta} = c\,\frac{\mathrm{d}k}{\mathrm{d}\beta}$: group velocity

Brioullin Diagram for Periodic Structures

$$\cos \beta d = \cos k\, d - (b/2)\sin kd$$

Fig. 3

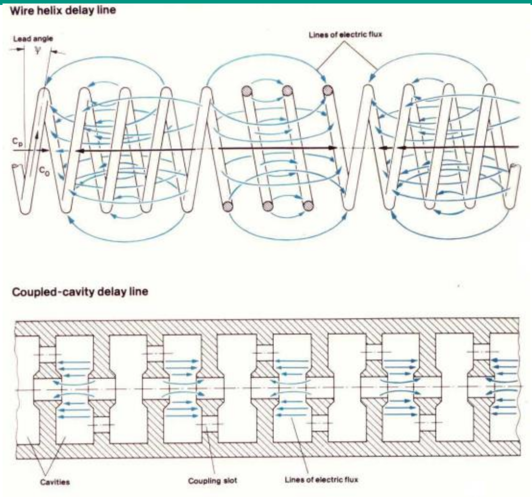


Typical dispersion curve of a non-radiating periodic structure

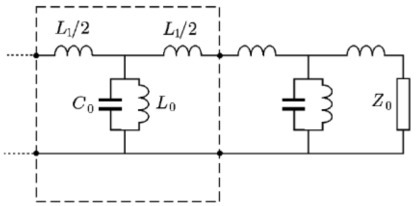
Passband and Stopband: These two situations of γ described above are called the passband and stopband. The frequency ranges corresponding to the solid lines in Fig.3 indicate the passband while the frequency regions without solid line correspond to the stopband.

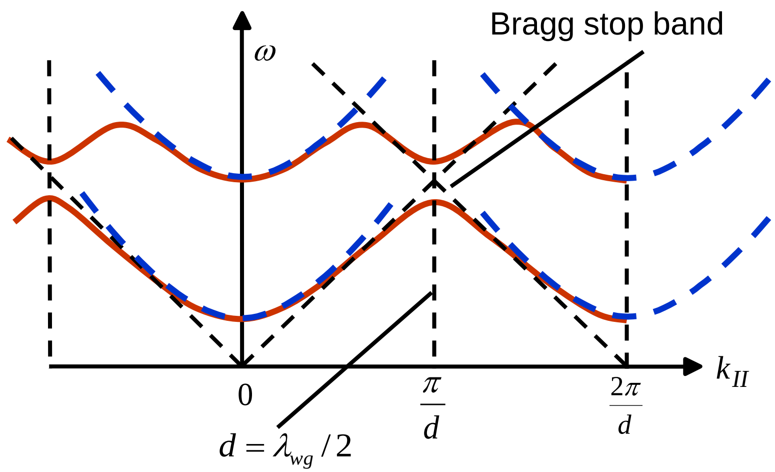
Slow Wave Effect: In the absence of periodic lumped elements, the propagation constant along the structure is given by k. Namely, $\beta = \beta_0 = k$. For a given k or kd, the value of β or βd on the dispersion curve of the periodic structure is always larger than β_0. Therefore, the phase velocity $v_p = kc/\beta$ (c: speed of light in vacuum) of the traveling wave in the periodic structure is slower than the free space speed of light. Hence, the periodic structure acts as a slow wave structure in its passband region. One important application is the delay line.

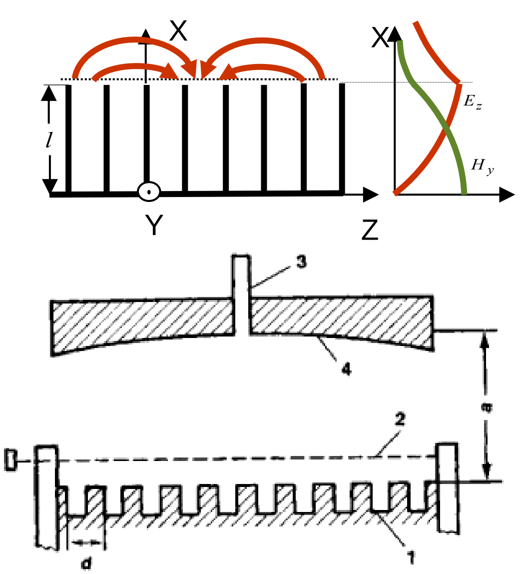
…, peridical structures for electron tubes



TWT
Travelling WaveTube







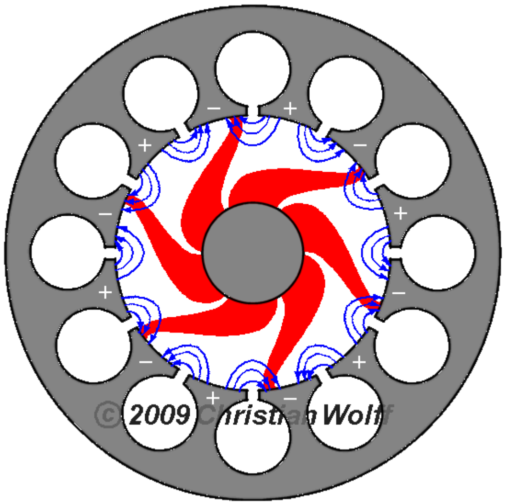
Orotron

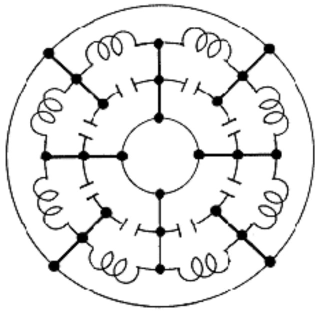
1. periodic structure,
2. electron flux,
3. radio wave guide,
4. mirror,

(a) distance between mirrors,
(d) period of the structure

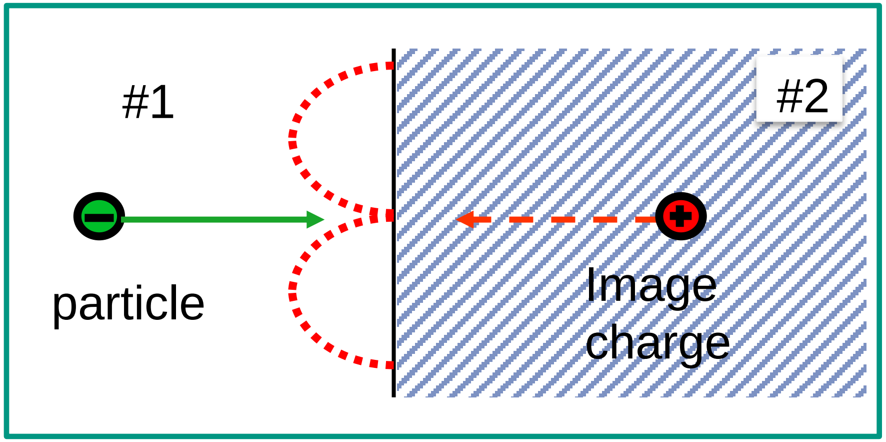
Magnetron

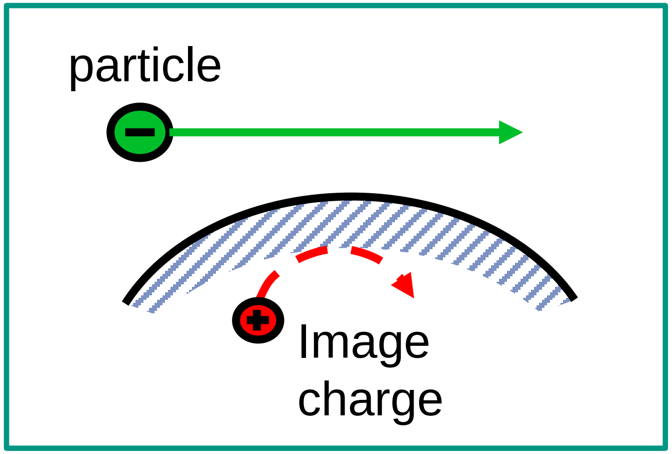
$\vec{v} \perp \vec{B}$





Transition Radiation





An electro magnetic wave will be emitted, if

an electron move from a medium in a different medium (with different ε)

an electron moves near a metallic surface/inhomogenity along.

Functional Principle:

Charged particle induced an image charge

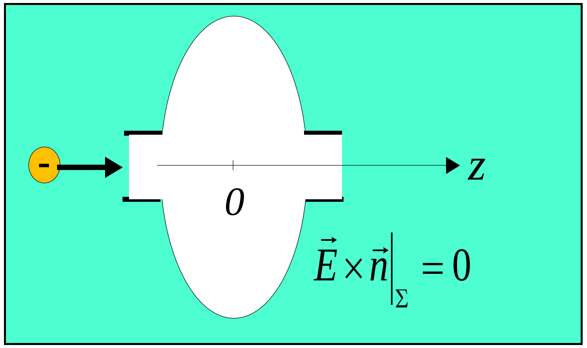
→ electrical dipole

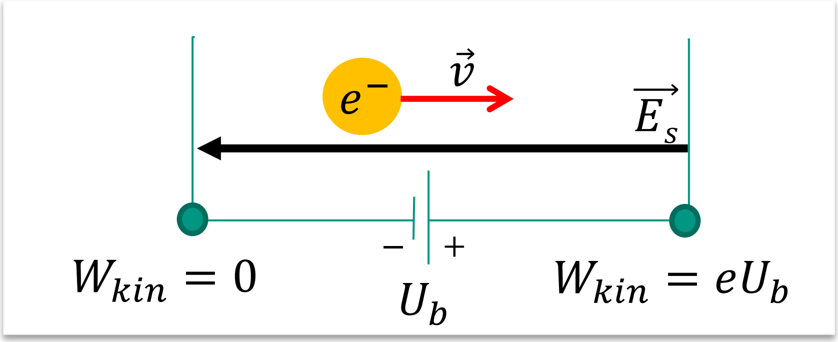
Particle crosses the boundary

→ change of the dipole field strength

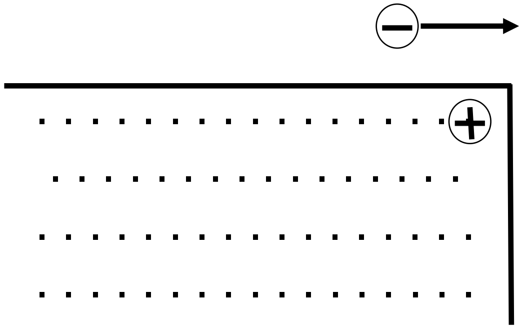
→ electro magnetic emission

… executed to the open resonator



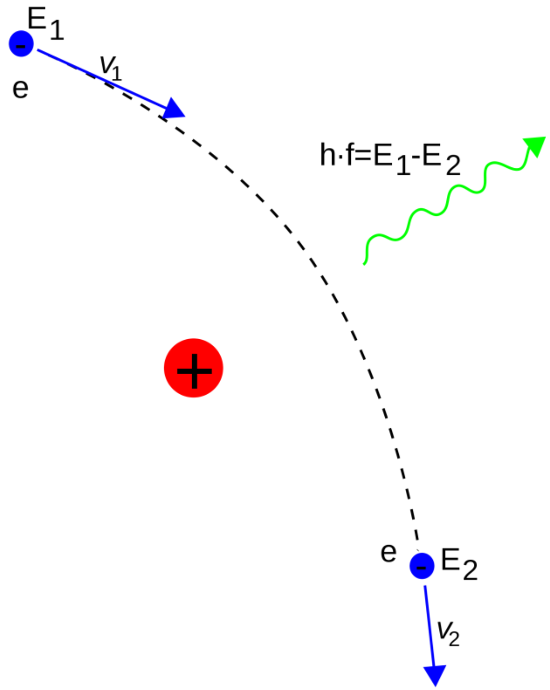


- Transition radiation is observed when a particle moves in a medium containing nonhomogeneities.
- For example, consider an electron passing near an angular nonhomogeneity.
  - The electron induces positive charge in metal or dielectric
  - Together with this charge forms a dipole that has a variable dipole moment and radiates.

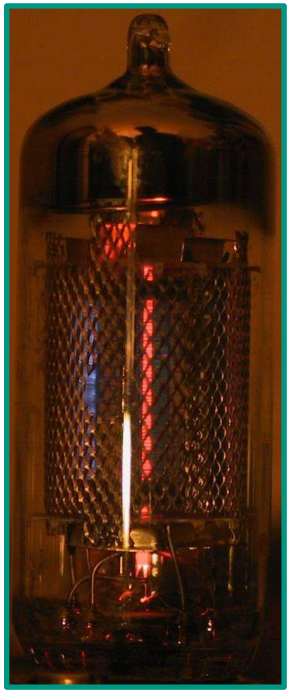


Transition Radiation on an edge non-homogeneity.

Bremsstrahlung



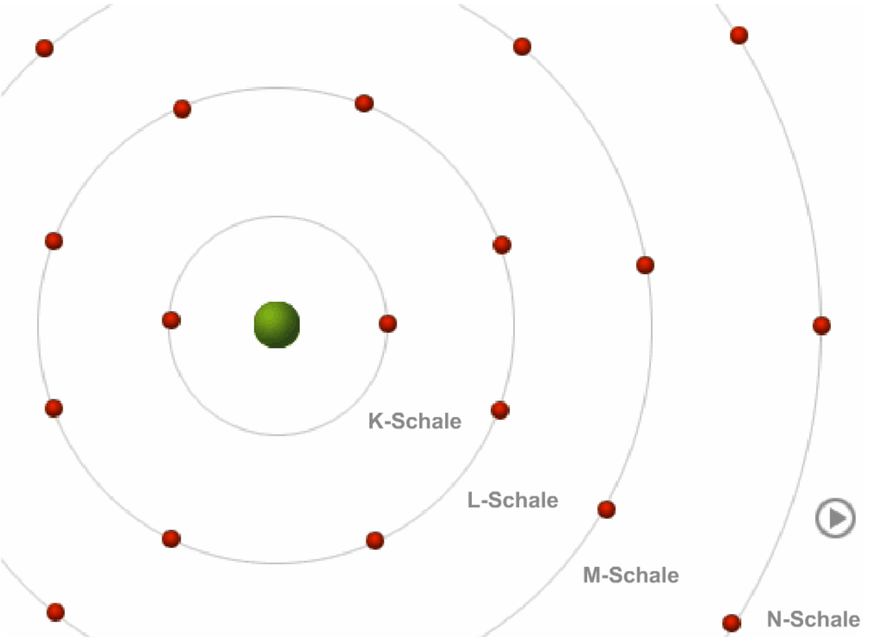
Generation of bremsstrahlung by acceleration of an electron in the Coulomb field of an atomic nucleus .



Bremsstrahlung (here exitation of metal ions) at the anode (EF89) und (PL95)
Source: wikipedia.org

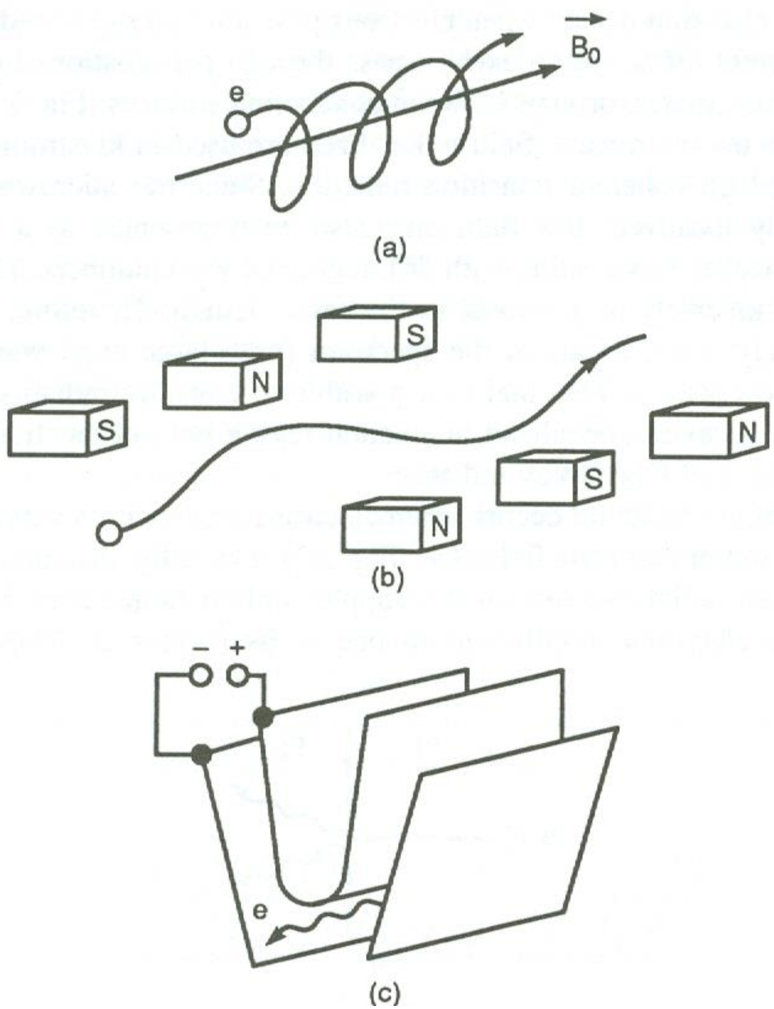
If an electric charge is accelerated, which means that the magnitude or the direction of the velocity is changed, electromagnetic radiation is generated. The energy of the radiation (photons) increases with increasing acceleration (or deceleration = negative acceleration).

Bremsstrahlung



Generation of bremsstrahlung by acceleration of an electron in the Coulomb field of an atomic nucleus .

Prospects of Bremsstrahlung



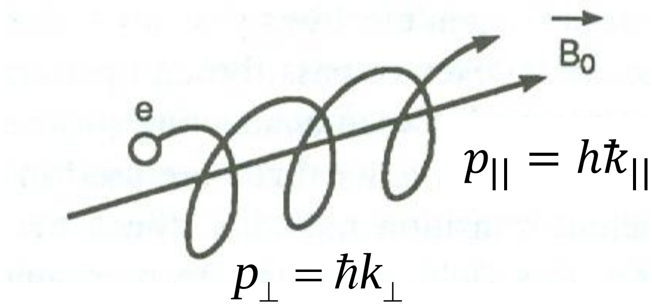
Quelle: V. Granatstein, Applications of High-Power Microwaves, Artech House, 1994

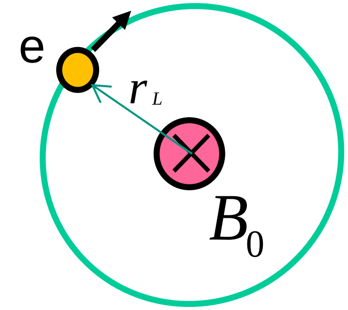
(a) Cyclotron and Synchrotron Radiation
in a constant magnetic field
→ Cyclotron Resonance Maser (CRM)

(b) Bremsstrahlung
in a periodic magnetic field
→ Free Electron Laser (FEL)

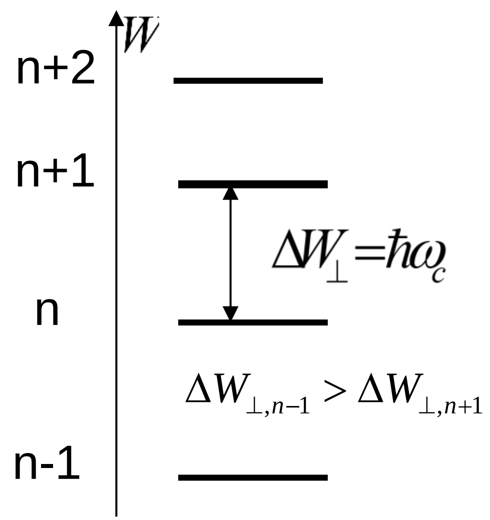
(a) Bremsstrahlung
in a electro static field
→ Vircator, Orbitron

Example: Electron Cyclotron Resonance





Quantum Energy Levels (Landau-Levels)

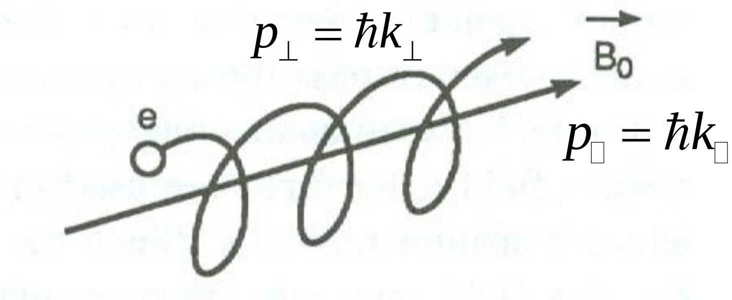


- Cyclotron resonance is related to the gyration of charged particles (electrons) around the field lines of a magnetic field.
- Electrons in a homogenous magnetic field B_0 have a discrete energy spectrum (Landau levels)

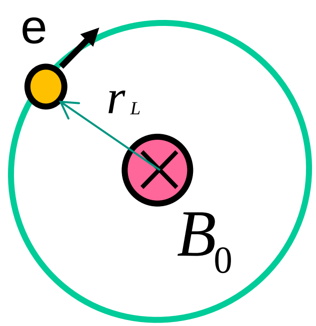
$$W_{\perp n} = mc^2\left[1+(2n+1)\hbar\omega_c/mc^2\right]^{1/2} - mc^2$$

- Electrons are able to absorb energy (transition: n → n+1) or radiate (transition n → n-1)
- It is essential for relativistic electrons that the quantum energy levels are not equally spaced.
- Since the probability of the transition n → n+1 is smaller than that of the transition n → n-1, there is a net energy transfer to the electromagnetic field.
- The radiated microwave has nearly the same frequency as the cyclotron frequency.

Example: Electron Cyclotron Resonance



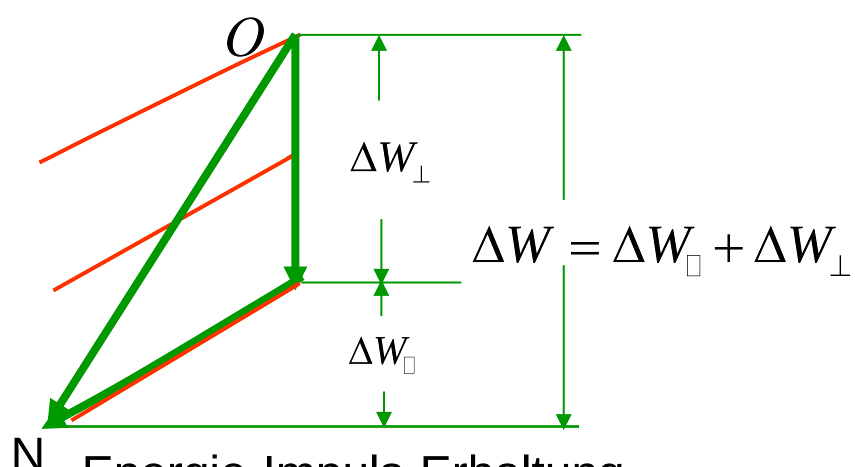
An electron, which moves on a helix around a constant magnetic field, has a transverse and axial.



$$\boxed{\omega_c = \frac{eB_0}{\gamma m_e} \approx 28 \cdot \frac{\mathrm{GHz} \cdot B_0/\mathrm{T}}{\gamma}}$$

$$\gamma = \frac{1}{\sqrt{1 - v_\perp^2 / c_0^2}}, \; p_\perp = \gamma m_e v_\perp$$

$$\boxed{r_L = \frac{v_\perp}{\omega_c} = \frac{p_\perp}{eB_0}}$$



Energie-Impuls-Erhaltung

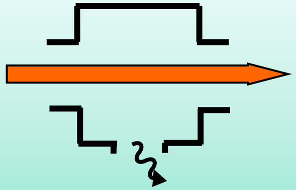
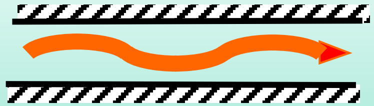
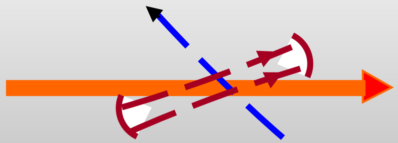
$$\Delta W_\perp = n \hbar \omega_c$$

$$\Delta W_\parallel = \left(\frac{\partial W}{\partial p_\parallel} \right) \Delta p_\parallel = v_\parallel \Delta p_\parallel = v_\parallel \hbar k_\parallel$$

$$\Delta W = \hbar \omega$$

$$\boxed{\omega = k_\parallel v_\parallel + n \omega_c}$$

Abstract

Electron Motion	Wave-Electron Synchronism	Interaction Scheme	RF Oscillators and Amplifier
$\dot{\vec{v}} = 0$	**Cherenkov** $v = v_{ph} \equiv \omega / k_{\parallel}$		TWT, BWO Magnetron
	transition $v = \omega / (k_{\parallel})_{Fourier}$		Monotron, Klystron
$\dot{\vec{v}} \neq 0$	**Bremsstrahlung** $\omega = k_{\parallel} v_{\parallel} + \Omega_c$		Gyrotron
	scattering $\dfrac{\omega_s - \omega_i}{k_{\parallel s} - k_{\parallel i}} = v$		Parametric devices